Ad soyad:

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MATRİSLER TEORİSİ ARA SINAV SORULARI

1. $\{(-1,3,1),(2,0,1)\}$ ve $\{(-1,0,1),(1,0,1),(2,1,1)\}$ kümeleri IR^3 reel vektör uzayı için bir baz oluşturur mu ? Elemanter işlemler yardımıyla araştırınız.

2. $A=\begin{bmatrix} 1 & 1 & 3 & 1 \\ 2 & 1 & 1 & 0 \\ 3 & 1 & 2 & 1 \end{bmatrix}$ matrisinin rankını elemanter işlemler yardımıyla bulunuz.

3. $B=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ matrisi veriliyor.

$f(t)=-t^3+6t^2-11t+6$ olduğuna göre $f(B)$ nedir?

4. $\begin{bmatrix} 1 & 3 & k \\ 3 & -2 & 5 \\ -4 & 5 & 4 \end{bmatrix}$ matrisinin simetrik matris olması için k ne olmalıdır?

5. $\begin{bmatrix} 2 & -3 \\ 1 & 3 \end{bmatrix}$ matrisini çarpanlarına ayırınız.

Başarılar Dilerim.

Süre 75 dk.

MATRİSLER TEO. ARASINAV CEVAPLARI

1) $A=\begin{array}{c} \begin{bmatrix} -1 & 3 & 1 \\ 2 & 0 & 1 \end{bmatrix} \end{array}\begin{array}{l} \cdots\alpha_1 \\ \cdots\alpha_2 \end{array} \overset{\varepsilon_1}{\sim} \begin{bmatrix} 1 & -3 & -1 \\ 2 & 0 & 1 \end{bmatrix} \overset{\varepsilon_2}{\sim} \begin{bmatrix} 1 & -3 & -1 \\ 0 & 6 & 3 \end{bmatrix} \overset{\varepsilon_3}{\sim} \begin{bmatrix} 1 & -3 & -1 \\ 0 & 1 & 1/2 \end{bmatrix}$

$\varepsilon_1 : \alpha_1 \to -\alpha_1$

$\varepsilon_2 : \alpha_2 \to \alpha_2 - 2\alpha_1$

$\varepsilon_3 : \alpha_2 \to \frac{1}{6}\alpha_2$

$\sim \begin{bmatrix} 1 & 0 & 1/2 \\ 0 & 1 & 1/2 \end{bmatrix} = R \begin{array}{l} \cdots\beta_1 \\ \cdots\beta_2 \end{array}$

$V_1 = Sp\{\alpha_1, \alpha_2\} = Sp\{\beta_1, \beta_2\} = V_2$, $boy\, V_2 = 2$ olup

$boy\, V_1 = 2$ dir. $\mathbb{R}^3$ için bir baz oluşturmaz. Benzer şekilde

$B = \begin{bmatrix} -1 & 0 & 1 \\ 1 & 0 & 1 \\ 2 & 1 & 1 \end{bmatrix} \begin{array}{l} -\alpha_1 \\ -\alpha_2 \\ -\alpha_3 \end{array} \sim \cdots\cdots \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = R \begin{array}{l} -\beta_1 \\ -\beta_2 \\ -\beta_3 \end{array}$

$boy\, V_1 = boy\, V_2 = 3$, $\mathbb{R}^3$ için bazdır. Burada

$V_1 = Sp\{\alpha_1, \alpha_2, \alpha_3\} = Sp\{\beta_1, \beta_2, \beta_3\} = V_2$

2) $$A=\begin{bmatrix} 1 & 1 & 3 & 1 \\ 2 & 1 & 1 & 0 \\ 3 & 1 & 2 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 3 & 1 \\ 0 & -1 & -5 & -2 \\ 0 & -2 & -7 & -2 \end{bmatrix} \sim \begin{pmatrix} 1 & 1 & 3 & 1 \\ 0 & 1 & 5 & 2 \\ 0 & -2 & -7 & -2 \end{pmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & -2 & -1 \\ 0 & 1 & 5 & 2 \\ 0 & 0 & 3 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -2 & -1 \\ 0 & 1 & 5 & 2 \\ 0 & 0 & 1 & 2/3 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 1/3 \\ 0 & 1 & 0 & -4/3 \\ 0 & 0 & 1 & 2/3 \end{bmatrix} = R$$

$\operatorname{rank} R = 3 = \operatorname{rank} A$

3) $f(B) = -B^3 + 6B^2 - 11B + 6I$

$$B^3 = (B.B).B = \left(\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}\right)\left(\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}\right)$$

$$= \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 9 \end{bmatrix}}_{B^2}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 27 \end{bmatrix}$$

$$f(B) = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -8 & 0 \\ 0 & 0 & -27 \end{bmatrix} + \begin{bmatrix} 6 & 0 & 0 \\ 0 & 24 & 0 \\ 0 & 0 & 54 \end{bmatrix} + \begin{bmatrix} -11 & 0 & 0 \\ 0 & -22 & 0 \\ 0 & 0 & -33 \end{bmatrix} + \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0$$

4) A simetrik $\Rightarrow A^t = A$

$$A^t = \begin{bmatrix} 1 & 3 & -4 \\ 3 & -2 & 5 \\ k & 5 & 4 \end{bmatrix} \qquad A = \begin{bmatrix} 1 & 3 & k \\ 3 & -2 & 5 \\ -4 & 5 & 4 \end{bmatrix} \Rightarrow k = -4$$

5) $$A = \begin{array}{c} \alpha_1 \\ \alpha_2 \end{array} \begin{bmatrix} -2 & -3 \\ -1 & 3 \end{bmatrix} \overset{\varepsilon_1}{\sim} \begin{bmatrix} 1 & 3 \\ 2 & -3 \end{bmatrix} \overset{\varepsilon_2}{\sim} \begin{bmatrix} 1 & 3 \\ 0 & -9 \end{bmatrix} \overset{\varepsilon_3}{\sim} \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} \overset{\varepsilon_4}{\sim} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = R$$

$\varepsilon_1 : \alpha_1 \leftrightarrow \alpha_2$

$\varepsilon_2 : \alpha_2 \to \alpha_2 - 2\alpha_1$

$\varepsilon_3 : \alpha_2 \to -\frac{1}{9}\alpha_2$

$\varepsilon_4 : \alpha_1 \to \alpha_1 - 3\alpha_2$

$$\varepsilon_4(\varepsilon_3(\varepsilon_2(\varepsilon_1 A))) = R$$

$$\Rightarrow A = \varepsilon_1^{-1}\varepsilon_2^{-1}\varepsilon_3^{-1}\varepsilon_4^{-1} R$$

$\varepsilon_1^{-1} : \alpha_1 \leftrightarrow \alpha_2$, $\varepsilon_2^{-1} : \alpha_2 \to \alpha_2 + 2\alpha_1$, $\varepsilon_3^{-1} : \alpha_2 \to -9\alpha_2$

$\varepsilon_4^{-1} : \alpha_1 \to \alpha_1 + 3\alpha_2$

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{array}{c} -\alpha_1 \\ -\alpha_2 \end{array}$$

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -9 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$